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ON CONJUGATED HEAT TRANSFER IN FULLY DEVELOPED FLOW

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NOMENCLATURE

- b, ratio of external and internal radii of the pipe, b'/a';
- C, $= 16E \times Pr$;
- E, Eckert Number;
- Pr, Prandtl Number;
- K, ratio of conductivities, K_2/K_1;
- k, ratio of diffusivities, k_2/k_1;
- K^*, conjugation parameter, $\sqrt{k}/K$;
- K_1^*, $= 1 + K^*$;
- K_2^*, $= (1 - K^*)/(1 + K^*)$;
- r, non-dimensional radial distance $= r'/a'$.

Greek symbols

- τ, non-dimensional time, $k_1 t/a'^2$;
- θ, non-dimensional temperature, $T - T_0/T_0$.

Subscripts

- 0, initial value;
- 1, fluid;
- 2, solid.

INTRODUCTION

SHANK and Dumore [1] extended the well known Graetz problem by taking into account the finite wall resistance. Recently some authors [2-5] have studied the effect of wall conduction on the conjugated heat transfer in channel flow.

The wall conduction effect becomes all the more important when the thickness of the pipe cannot be discarded in comparison with its radius and also when the flux of heat is high; a situation particularly encountered in the design of compact heat exchangers, concentrating upon the small passage size and the increase in heat transferred per unit of pumping power. Design problems in nuclear reactors also demand the knowledge of temperature distribution in the fluid, in which heat is being generated, immediately after the energy is released.

This study deals with unsteady heat transfer to fully developed flow in a heat conducting pipe of finite thickness when the outer periphery of pipe undergoes a step change in heat flux or surface temperature.

Equations for the temperature distribution in the composite regions of the fluid and solid coupled through matching boundary conditions at the interface, are solved by the method of Laplace transform, yielding series solutions valid for small time periods after the transition has occurred.

For the problem stated the equations in the non-dimensional form are:

energy equation for fluid [6]

$$\frac{\partial \theta_1}{\partial \tau} = \frac{\partial_2^2 \theta_1}{dr^2} + \frac{1}{r}\frac{\partial \theta_1}{\partial r} + Cr^2, \quad 0 \le r \le 1 \tag{1}$$

conduction equation for solid

$$\frac{\partial \theta_2}{\partial \tau} = k\left(\frac{\partial^2 \theta_2}{dr^2} + \frac{1}{r}\frac{\partial \theta_2}{dr}\right), \quad 1 \le r \le b \tag{2}$$

with initial and boundary conditions

$$\theta_1 = \theta_2 = 0 \quad \text{for } \tau \le 0, \quad 0 \le r \le b \tag{3a}$$

and for $\tau > 0$

$$\theta_1 = \theta_2, \quad \text{at } r = 1 \tag{3b}$$

$$\frac{\partial \theta_1}{dr} = K\frac{\partial \theta_2}{dr}, \text{ at } r = 1 \tag{3c}$$

and

$$\frac{\partial \theta_2}{\partial r} = S_1 \text{ (const.), at } r = b \tag{3d}$$

or

$$\theta_2 = S_2 \text{ (const.), at } r = b. \tag{3e}$$

The solutions of equations (1) and (2) under conditions (3) are obtained by applying Laplace transform technique. As these solutions involve Bessel functions of second kind it is unwieldy to get their inversions as such. Following Carslaw and Jaeger [7] we expand the functions for large values of the Laplace's parameter and obtain term by term inversion of the series. This yields results valid for small values of time.

Thus the nondimensional temperature distribution in the fluid and the solid regions are respectively given by:

Case (i). Step change in heat flux

$$\theta_1 = \sum_{n=0}^{\infty} \frac{K_2^{*n}}{K_1^* \sqrt{r}} \Bigg\{ 2S_1 \sqrt{bk}\,(4\tau)^{1/2} A_1 - 2CK^* \sum_{u=0}^{1} (4\tau)^{3/2} A_3 - C \sum_{u=0}^{1} (-1)^u (4\tau) \left[A_2 + 16\,\tau A_4 \right] \Bigg\} + Cr^2\tau + 2C\tau^2 \tag{4}$$

and

$$\theta_2 = \sum_{n=0}^{\infty} \frac{K_2^{*n}}{K_1^* \sqrt{r}} \Bigg\{ K_1^* S_1 \sqrt{bk} \sum_{v=0}^{1} K_2^{*v} (4\tau)^{1/2} B_1 - 2CK^* \sum_{w=0} (4\tau)^{3/2} B_3$$

$$+ 4\tau CK^* \sum_{w=0}^{1} [B_2 + 16\tau B_4]\bigg\}. \tag{5}$$

Case (ii). Step change in surface temperature

$$\theta_1 = \sum_{n=0}^{\infty} \frac{(-1)^n K_2^{*n}}{K_1^* \sqrt{r}} \bigg\{ K_1^* S_2 \sqrt{b}\, \mathrm{erfc}(Z_1) - 2CK^* \sum_{u=0}^{1} (-1)^u.(4\tau)^{3/2} A_3 - C \sum_{u=0}^{1} 4\tau \times [A_2 + 16\tau A_4] \bigg\} + Cr^2\tau + 2C\tau^2 \tag{6}$$

$$\theta_2 = \sum_{n=0}^{\infty} \frac{(-1)^n K_2^{*n}}{K_1^* \sqrt{r}} \cdot \bigg[K_1^* S_2 \sqrt{b} \sum_{v=0}^{1} K_2^{*v} \mathrm{erfc}(Z_2) - 2CK^* \sum_{w=0}^{1} (-1)\tau^{3/2} B_3 + 4C\tau K^* \sum_{w=0}^{1} (B_2 + 16\tau B_4) \bigg] \tag{7}$$

where A's, B's are known functions of physical parameters of the problem and are given by

$$A_l = i^l \mathrm{erfc}(Z_1/2\sqrt{k\tau}),$$
$$B_l = i^l \mathrm{erfc}(Z_2/2\sqrt{k\tau}),$$
$$Z_1 = b_{nr} \text{ for } l = 1 \text{ and } b_{nru} \text{ for } l = 2, 3, 4,$$
$$Z_2 = b_{nrv} \text{ for } l = 1 \text{ and } b_{nrw} \text{ for } l = 2, 3, 4,$$
$$b_{nr} = (2n+1)(b-1) - \sqrt{k}(r-1),$$
$$b_{nru} = 2(n+u)(b-1) - \sqrt{k}(r-1),$$
$$b_{nrv} = (2n+1)(b-1) - (-1)^v(r-1),$$
$$b_{nrw} = (2n+1)(b-1) - (-1)^w(b-r).$$

DISCUSSION

Equation (4) gives the contribution of conductional and frictional heat towards the development of temperature field in the fluid region. Terms occurring with S_1 are due to conduction and are the same as for the fluid at rest, whereas those occurring with C are caused by friction. These having been denoted by ϕ and ψ respectively, the values of ϕ/S_1 and ψ/C have been depicted for different values of τ in Fig. 1. The former have their maximum value near the surface $r = 1$. This is due to dominance of conductance near the wall. The later can be seen to have their maximum around $r = 0.8$ a little way from the wall (in the fluid region) which, for the parameters considered, may be a region of highest velocity gradient in the fully developed boundary layer. Sample curves have been drawn for Case (i) only. Similar curves can however be drawn for Case (ii) also.

Table 1 shows the dependence of the interfacial temperature on the conjugation parameter K^*, which is the symbolic representative of the diffusive and conductive properties of both the regions. The temperature is seen to decrease with the increase in K^* for the parameters $\sqrt{k} = 2$, $b = 1.2$, $C = 1$ and $S_1 = 1$.

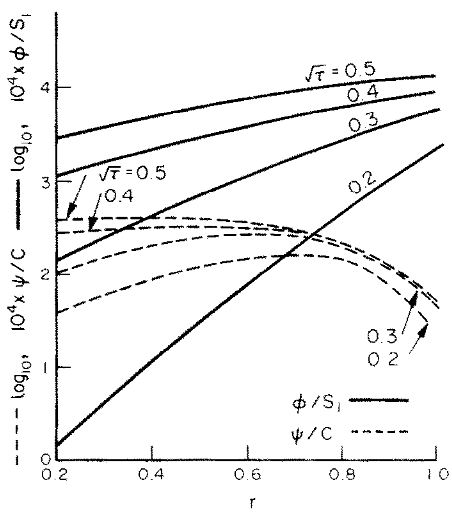

FIG. 1. θ_1 at $r = 1$.

Table 1. Variation of interfacial temperature, θ_1 $\tau = 1$ with K^* for different values of τ

$\sqrt{\tau}$	K^* 0.0005	0.005	0.05	0.5	5
0.1	0.204	0.198	0.164	0.128	0.062
0.3	1.954	1.923	1.646	0.841	0.284
0.5	5.204	4.723	4.286	1.624	0.485

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